CHAPTER

2 Systems of Linear Equations and Inequalities

Ticket vendors often charge different prices for tickets to the same event, depending on the age of the attendee or the time left before the event. You will use systems of linear equations and linear inequalities to model the income for an event with different ticket prices.

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2.1 Finding a Job

Introduction to Systems of Linear Equations

Objectives

In this lesson, you will

- Solve a system of linear equations.
- Interpret the solution to a system of linear equations.

Key Terms

- system of linear equations
- point of intersection

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Problem 1

A friend of yours interviewed for two different sales positions in competing companies. Stellar Sales pays $500 per week plus 10% commission on the total dollars each individual sells per week. Lunar TeleSales pays $200 per week but offers a 20% commission on the total dollars each individual sells per week. Sales at both companies are seasonal.

For **Stellar Sales** there are two quantities that are changing:

1. Define a variable for each of these quantities.

2. Identify which variable is the independent variable.

3. Identify which variable is the dependent variable.

4. Write an equation that shows the relationship between these variables.

Use your equation to answer the following questions.

5. Calculate your friend's weekly pay working for Stellar Sales if his sales were $1000 for one week. What if his sales were $2000 for one week?

6. If his total compensation with Stellar Sales for one week is $625, what were his sales for the week?

7. If his total compensation with Stellar Sales for one week is $727, what were his sales for the week?

For **Lunar TeleSales** there are two quantities that are changing:

8. Define a variable for each of these quantities.

9. Identify which variable is the independent variable.

10. Identify which variable is the dependent variable.

11. Write an equation that shows the relationship between these variables.

Use your equation to answer the following questions.

12. Calculate your friend's weekly pay working for Lunar TeleSales if his sales were $1000 for one week. What if his sales were $2000 for one week?

13. If his total compensation with Lunar TeleSales for one week is $625, what were his sales for the week?

14. If his total compensation with Lunar TeleSales for one week is $727, what were his sales for the week?

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15. In both of these situations, you have constructed a linear model for each of the companies. How do you know they are linear?

16. For each model, identify the slope and *y*-intercept and explain what each means in the problem situation.

Stellar Sales

Slope: _______

y-intercept: ______

Lunar TeleSales

Slope: _______

y-intercept: _______

To determine which option is best, gather some data by answering the following questions.

17. If the sales in a particular week total $1200, what salary will your friend receive from:

a. Stellar Sales

b. Lunar TeleSales

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18. Your friend's weekly earnings from Stellar Sales are $1200. If his weekly sales were the same, what would his weekly earnings be from Lunar TeleSales?

19. Your friend's weekly earnings from Lunar TeleSales are $540. If his weekly sales were the same, what would his weekly earnings be from Stellar Sales?

20. Suppose two salespeople, one at each company, had the same weekly sales. Would their weekly earnings ever be equal? If so, indicate the weekly sales and earnings.

A **system of linear equations** is two or more linear equations.

21. Use the information gathered to complete the following table, and graph both linear functions in the grid.

Quantity Name			
Unit			
Expression			

Using the graph, answer the following questions:

22. At what point do the two lines intersect?

23. In this problem situation, explain what is represented by the point of intersection. What is represented by the points to the left of this point and the points to the right?

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Be prepared to share your work with another pair, group, or the entire class.

2.2 Pens-R-Us
Solving Systems of Linear Equations: Graphing and Substitution

Objectives

In this lesson, you will

- Solve systems of linear functions graphically.
- Solve systems of linear functions using substitution.

Key Term

- substitution

2

Problem 1

A company that manufactures inexpensive pens and sells them in bulk wants to launch a new model that has a comfort grip. They have estimated that in order to start a new line, there are some fixed costs including new machinery, labor costs, design costs, and overhead, which amount to about $22,000. There are also material costs for each individual unit, which are estimated to be $0.075 per unit. Based on previous models, they have decided to sell these pens for $11.25 per 100 units. Answer the following questions.

1. Define variables for the quantities that are changing in the manufacturing of the pens and write an equation for the cost to manufacture the new line of pens.

Use your equation to answer the following questions.

2. How much would it cost to manufacture 1000 pens? 50,000 pens? 100,000 pens?

3. If they have budgeted $50,000 for these new pens, how many pens can they manufacture?

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4. Define variables for the quantities that are changing in the selling of the pens, and write an equation for the income from the sale of the new line of pens.

Use your equation to answer the following questions.

5. How much income will the company receive if they sold 1000 pens? 50,000 pens? 100,000 pens?

6. If the company received $100,000 from the sales of these new pens, how many pens were sold?

7. For each of these functions, identify the slope and *y*-intercept and explain what each means in the problem situation.

Cost to manufacture.

Slope: _______

y-intercept: _______

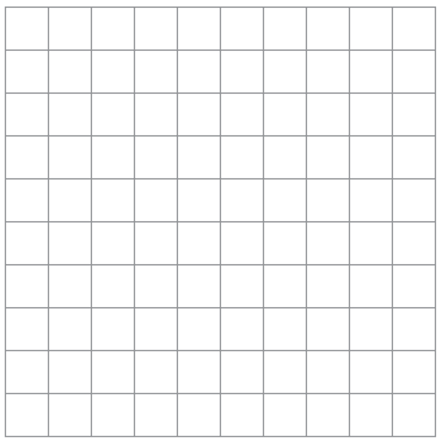
Income from sales.

Slope: _______

y-intercept: ___

Use the information gathered to complete the following table, and graph both linear functions in the grid.

Quantity Name			
Unit			
Expression			



8. Use your graph to determine the point of intersection of the two lines. Is this an exact answer? Explain.

9. In this problem situation, explain what is represented by the point of intersection. What is represented by the points to the left of this point and the points to the right?

Another method for determining the point of intersection is algebraic **substitution**. In this method:

A. Solve one equation for one of the variables

B. Substitute that expression for the variable in the second equation

C. Solve for the variable that is left, then

D. Substitute this value into either of the equations to calculate the value for the other variable, and finally

E. Substitute both values into both equations to check your work.

10. Use algebraic substitution to determine the exact point of intersection.

Point of intersection: ____________________

11. Does this answer make sense in the problem situation? Explain.

12. Determine the domain and range of the following:

the cost to manufacture the pens

the income from selling the pens

13. Are the range and domain the same as in the algebraic equations?

For each of the following systems of linear equations use the slope and intercept to graph the equations and determine the point of intersection.

14. $\begin{cases} y = 2x - 7 \\ y = -3x + 3 \end{cases}$ Point of intersection: ________

15. $\begin{cases} y = -2.5x - 10 \\ y = -2.4x + 10 \end{cases}$ Point of intersection: ____________

16. Use algebraic substitution to check your answers from Question 15.

a. $\begin{cases} y = 2x - 7 \\ y = -3x + 3 \end{cases}$

Point of intersection: ______

b. $\begin{cases} y = -2.5x - 10 \\ y = -2.4x + 10 \end{cases}$

Point of intersection: __________

Use algebraic substitution to determine the points of intersection for each system of linear equations.

17. $\begin{cases} y = 4x - 3 \\ 4x + 5y = 9 \end{cases}$

Point of intersection: _____

18. $\begin{cases} 3x + 2y = 8 \\ 2x - y = 3 \end{cases}$

Point of intersection: ______

19. $\begin{cases} y = -x - 9 \\ x = 3y - 8 \end{cases}$

Point of intersection: ______________

20. $\begin{cases} y = 2.3x - 3.7 \\ y = 8.3x + 5.3 \end{cases}$

Point of intersection: ____________

Be prepared to share your work with another pair, group, or the entire class.

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2.3 Tickets
Solving Systems of Linear Equations: Linear Combinations

Objective
In this lesson, you will
- Solve systems of linear equations using linear combinations.

Key Term
- linear combinations or elimination

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Problem 1

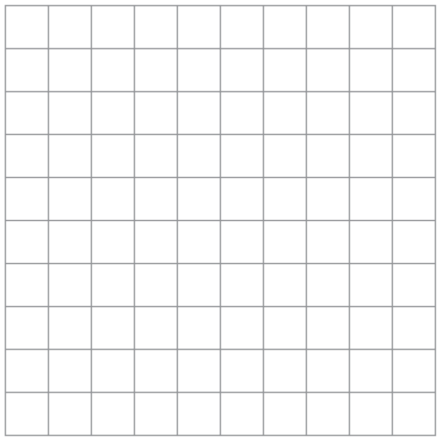
For the fall play, *Othello*, the thespian club sold student tickets and adult tickets. For Friday night, they sold 125 adult tickets and 325 student tickets for a total of \$2212.50. For Saturday night, they sold 415 student tickets and 250 adult tickets for a total of \$3367.50. If we want to determine the price of a student ticket and the price of an adult ticket, we need to solve a system of linear equations in standard form.

1. In this problem, there are quantities that are changing; define variables for these quantities, and write an equation for the total income for Friday night.

2. Using the same variables, write an equation for the total income for Saturday night.

3. Is one of these variables dependent? Explain.

4. One way to solve this system would be to graph the equations using the *x*- and *y*-intercepts to determine the point of intersection. Use the grid to graph these equations and determine the point of intersection.



5. Using this graph, were you able to determine an exact point of intersection? Explain.

Problem 2

A second way would be to use substitution. First solve one of the equations for one of the variables. Then substitute the expression into the other equation, and solve the resulting one-variable equation. Substitute that solution back into one of the original equations to calculate the value of the second variable. Then check your work by substituting the value of the second variable into each equation.

1. Use this method to solve this system.

Point of intersection: _________

2. Was this easy or hard? Explain.

A third method, called **linear combinations** or **elimination**, often reduces the level of difficulty, especially when you are working with linear equations in standard form. Remember that when you solved linear equations in one variable, you were able to perform four transformations on an equation without changing the value of the variables. Those transformations enabled you to add, subtract, multiply, and divide equal quantities to both sides of an equation.

3. So, let's begin by rewriting the original equations as a system:

4. Next, examine the coefficients of each variable; do you notice a relationship between these coefficients?

5. a. How could you use addition, subtraction, multiplication, or division to transform the system so that the coefficients of one of the variables are the same? Could you do the same for the other variable? Explain.

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b. Transform the system so that the coefficients of one variable are the same.

6. Since the sides of an equation represent equal quantities, which transformation could you perform on this system to eliminate one of the variables? Perform this transformation and write the result.

7. Solve this equation for the remaining variable.

8. Substitute this value back into one of the original equations, and solve for the remaining variable.

9. Substitute both values back into the other original equation to check that your answers are correct.

This method of solving systems of linear equations is called linear combinations because you perform various linear transformations and then combine the resulting equations. Repeat this process for each of the following systems of equations:

10. $\begin{cases} -2x + 3y = 11 \\ 5x + y = -2 \end{cases}$

Point of intersection: ________

11. $\begin{cases} 4x - 3y = 11 \\ 2x + 5y = -1 \end{cases}$

Point of intersection: ________

12. $\begin{cases} -7x + 3y = -23 \\ 3x + 5y = -9 \end{cases}$

Point of intersection: ________

13. $\begin{cases} -5x + 4y = 13 \\ 6x + 7y = 8 \end{cases}$

Point of intersection: ________

14. $\begin{cases} 75x + 325y = 250 \\ -6x + 5y = 11 \end{cases}$

Point of intersection: ________

15. $\begin{cases} -2.5x + 3.6y = 2.2 \\ 2x + 9y = 22 \end{cases}$

Point of intersection: ______

16. $\begin{cases} \dfrac{-1}{3}x + \dfrac{5}{6}y = 3 \\ \dfrac{3}{2}x + \dfrac{1}{3}y = 11 \end{cases}$

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Point of intersection: ______

Be prepared to share your work with another pair, group, or the entire class.

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2.4 Cramer's Rule

Solving Systems of Linear Equations: Cramer's Rule

Objectives

In this lesson, you will

- Write and evaluate determinants.
- Use Cramer's Rule to solve systems of linear functions in two variables.

Key Terms

- Cramer's Rule
- square array
- determinant

Problem 1

A Swiss mathematician who was a professor of mathematics in Geneva, Gabriel Cramer (1704–1752) found an additional method for solving systems of linear equations. When Cramer published his rule in 1750, he did not use the methods as they are now shown, and he gave no explanation for how he achieved the result. A more thorough development and explanation of this process can be found at www.carnegielearning.com/Cramersrule. This procedure, thereafter called **Cramer's Rule**, can also be applied to systems of linear equations in three or more variables.

Cramer's procedure relies on an algebraic operation that transforms a **square array** of numbers into a single value. This operation is called calculating the value of the **determinant** of the square array. A square array of numbers is written in this form:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

In a square array, a_{ij} means the number in the ith row and jth column.

To solve a system of linear equations in two variables, use a 2×2 array:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The formula for calculating the value of the determinant of a 2×2 array is

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Or using simpler notation:

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Calculate the values of the following determinants.

1. $\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} =$

2. $\begin{vmatrix} -2 & 6 \\ -5 & 7 \end{vmatrix} =$

3. $\begin{vmatrix} 1 & -6 \\ 3 & 7 \end{vmatrix} =$

4. $\begin{vmatrix} 30 & 50 \\ -25 & 45 \end{vmatrix} =$

5. $\begin{vmatrix} 3.3 & 5.2 \\ 2.4 & -4.1 \end{vmatrix} =$

6. $\begin{vmatrix} \frac{3}{2} & \frac{5}{6} \\ \frac{2}{3} & \frac{4}{5} \end{vmatrix} =$

Using Cramer's Rule to solve a system of linear equations in two variables requires defining some 2 × 2 determinants and calculating their values. To create these arrays, the equations must be in standard form. The determinants are defined as follows:

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$ where *a, b, c, d, e,* and *f* are constants and *x* and *y* are variables

$$D = \begin{vmatrix} a & b \\ d & e \end{vmatrix} \qquad D_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix} \qquad D_y = \begin{vmatrix} a & c \\ d & f \end{vmatrix}$$

To determine the solution you then calculate these ratios:

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad \text{where } D \neq 0$$

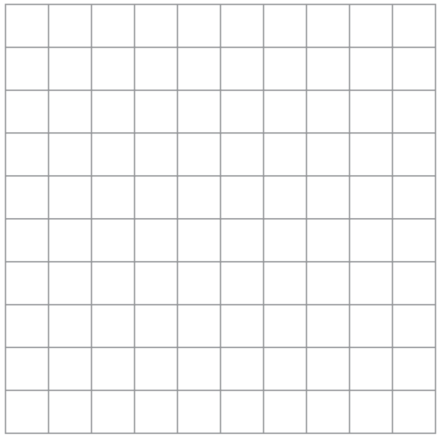
Notice that D is just the determinant of the array coefficients of the variables, D_x is D with the coefficients of x replaced with the answers column, and D_y is D with the coefficients of y replaced with the answers column.

Once you calculate values for x and y, substitute the values into each equation to check your work.

Solve each of the following systems of equations in three ways: graphically, using linear combinations, and using Cramer's Rule. Make sure to show all your work.

7. $\begin{cases} 2x + y = 6 \\ -x + 2y = 2 \end{cases}$

a. Graphically



Point of intersection: ______

b. Linear Combinations

Point of intersection: ______

c. Cramer's Rule

Point of intersection: ______

For each of the following systems of equations use Cramer's Rule.

8. $\begin{cases} 5x - 2y = 0 \\ 4x - 3y = -7 \end{cases}$

Point of intersection: ______

9. $\begin{cases} x - y = 6 \\ -x + 3y = 0 \end{cases}$

Point of intersection: ______

10. $\begin{cases} 5x + y = 17 \\ -3x - 5y = -41 \end{cases}$

Point of intersection: ______

11. $\begin{cases} 2.2x + 4.7y = 6.3 \\ -1.2x + 3.2y = -9.2 \end{cases}$

(Hint: What makes this system hard? Can you do something to make it easier?)

Point of intersection: ______

12. $\begin{cases} \frac{2}{3}x + \frac{1}{5}y = 3 \\ \frac{3}{2}x + \frac{5}{3}y = \frac{2}{3} \end{cases}$

(Hint: What makes this system difficult to solve? Can you do something to make it easier?)

Point of intersection: ______

Be prepared to share your work with another pair, group, or the entire class.

2.5 Consistent and Independent

Systems of Linear Equations: Consistent and Independent

Objectives

In this lesson, you will

- Solve systems that are linearly independent and dependent.
- Solve systems that are consistent and inconsistent.

Key Terms

- consistent
- inconsistent
- linearly independent
- linearly dependent

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Problem 1 Which Method When?

In this chapter, we have found four different methods for solving systems of linear equations:

- Graphing
- Substitution
- Linear Combinations/Elimination
- Cramer's Rule/Determinants

1. Which of these methods do you prefer? Explain.

2. Which do you least like to use? Explain.

3. Which is the least accurate? Explain.

Each of these methods has its advantages and disadvantages, so deciding which method to use to solve a particular system can be based on the form of the equations, the numbers involved, or just which method you find more efficient or easiest to use.

For each of the following systems, choose a method and determine the solution.

4. $\begin{cases} 2x - 3y = 9 \\ -4x + 3y = 5 \end{cases}$

Point of intersection: __________

5. $\begin{cases} y = x - 6 \\ -x + 3y = -2 \end{cases}$

Point of intersection: _____

6. $\begin{cases} 4x - 5y = 6 \\ -7x + 3y = 12 \end{cases}$

Point of intersection: ___________

7. $\begin{cases} 2x - 5y = 12 \\ -3x + 4y = -25 \end{cases}$

Point of intersection: ______

Be prepared to share your work with another pair, group, or the entire class.

Problem 2 What Happened Here?

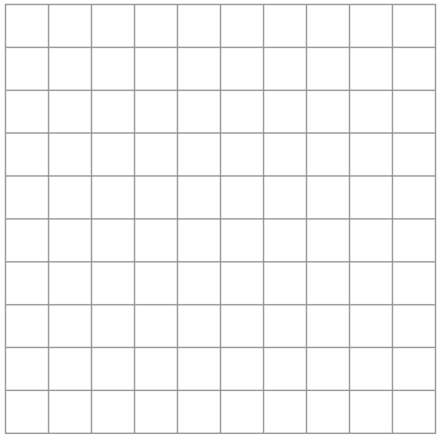
1. Solve the following system of linear equations using linear combinations:

$$\begin{cases} 2x - 5y = 12 \\ -4x + 10y = -25 \end{cases}$$

Point of intersection: ______________________________

a. What happened when you added the equations? Why?

b. Graph both of the lines in the grid.



c. What is the relationship between the lines? Why?

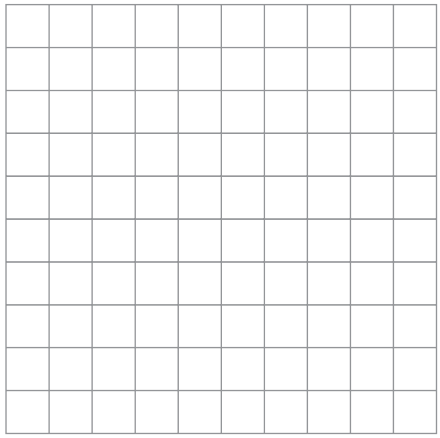
2. Solve the following system of linear equations using substitution:

$$\begin{cases} y = x + 4 \\ -2x + 2y = 8 \end{cases}$$

Point of intersection: ____

a. What happened when you solved the system? Why?

b. Graph both of the lines in the grid.



c. What is the relationship between the lines? Why?

3. How were the solutions to the first four systems in Problem 1 different from the last two systems in Problem 2?

When a system of equations has at least one solution, the system is said to be **consistent**. If a system has no solutions, it is said to be **inconsistent**.

4. Which of the six systems of equations in Problems 1 and 2 are consistent? Inconsistent?

5. When is a system of equations inconsistent? Explain.

When a system of equations has one solution or no solutions, the system is said to be **linearly independent**, and when the system has an infinite number of solutions, it is said to be **linearly dependent**.

6. Which of the six systems of equations in Problems 1 and 2 are linearly independent? Linearly dependent? Explain.

7. Go back and label each system in Problems 1 and 2 according to consistency and independency.

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For each of the following systems, choose a method, determine its solution, and label it according to the system's consistency and independency.

8. $\begin{cases} 8x - 12y = 7 \\ -4x + 6y = 5 \end{cases}$

Method:

Point of intersection: ____________________

9. $\begin{cases} y = 2x - 3 \\ -2x + 3y = 8 \end{cases}$

Method:

Point of intersection: ________

10. $\begin{cases} y = 3x - 6 \\ -7x + 3y = -10 \end{cases}$

Method:

Point of intersection: ______

11. $\begin{cases} y = -3x + 12 \\ 6x + 2y = 24 \end{cases}$

Method:

Point of intersection: ______________________________

Be prepared to share your work with another pair, group, or the entire class.

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2.6 Inequalities–Infinite Solutions

Solving Linear Inequalities and Systems of Linear Inequalities in Two Variables

Objectives

In this lesson, you will

- Solve linear inequalities in two variables.
- Solve systems of linear inequalities in two variables.

Key Terms

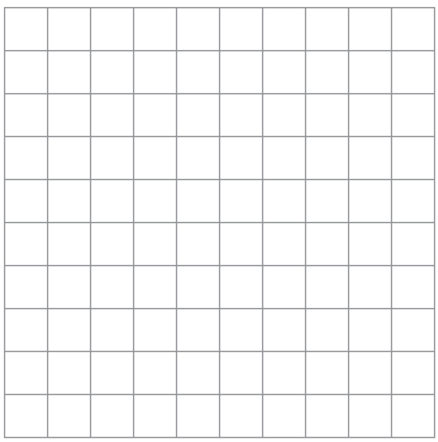
- linear inequalities in two variables
- half-plane
- systems of two linear inequalities in two variables

Problem 1

Previously, you solved linear inequalities in one variable and found that there were an infinite number of solutions. To show your solution, you used a number line. In fact, the solution was a segment on the number line or a ray with or without end points. When working with **linear inequalities in two variables**, there are also infinite solutions; however, the solutions are not numbers but ordered pairs. So, the solution to a linear inequality in two variables is an infinite number of ordered pairs.

1. Graph the equation in the grid.

$y = 2x + 3$



a. Write two ordered pairs that satisfy this equation.

b. Demonstrate how you can tell if these ordered pairs are solutions using the equation. Also explain how you can use the graph of the line to tell if these ordered pairs are solutions.

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2. Choose an ordered pair that does not satisfy the equation $y = 2x + 3$. Graph this ordered pair in the grid.

a. Demonstrate how you can tell whether the ordered pair in Question 2 is a solution using the equation. Also explain how you can use the graph of the line to tell if these ordered pairs are solutions.

b. Rewrite the original equation as an inequality using $<$ or $>$ so this ordered pair would be a solution of the inequality.

c. Is this the only ordered pair that would satisfy the inequality that you wrote?

d. Describe the position on the graph of the other ordered pairs that satisfy your inequality.

e. Shade the graph to indicate all of the solutions to the inequality.

f. Do the points on the line satisfy the inequality? Are they included in the solution set for this inequality? Explain.

g. The solution set for linear inequalities is called a **half-plane**. Why?

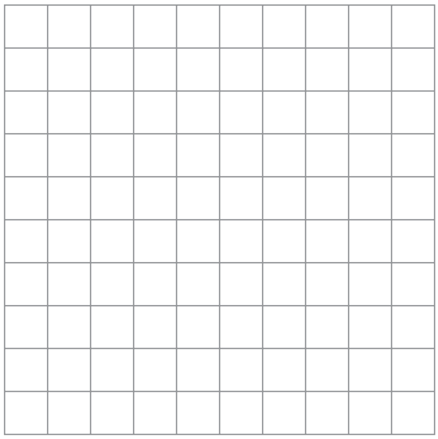
h. If the inequality uses $<$ or $>$, will the line be included in the solution set? Explain.

Solve each of the following inequalities by graphing their solution sets. Perform these steps to determine which half-plane is the solution:

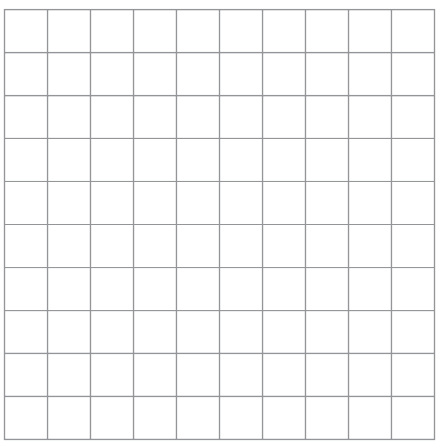
- Choose a point on one side of the line
- Substitute the ordered pair for this point into the inequality
- If this ordered pair satisfies the inequality, then all the other points on that side of the line are solutions, but
- If it does not satisfy the inequality, then all the points on the other side of the line are solutions.

Remember, if the line is included, use a solid line (———), and if the line is not included, use a dashed line (— — — -).

3. $y \leq -3x + 1$



4. $y > \frac{1}{2}x - 2$



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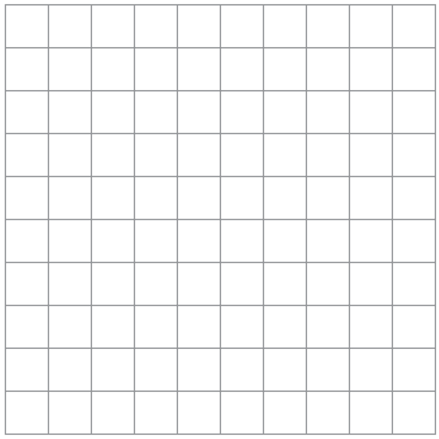
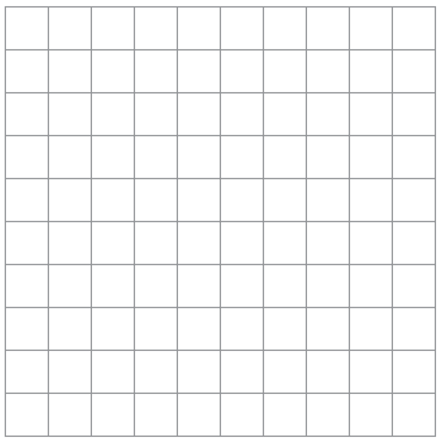
5. $-3x + 2y \geq 6$



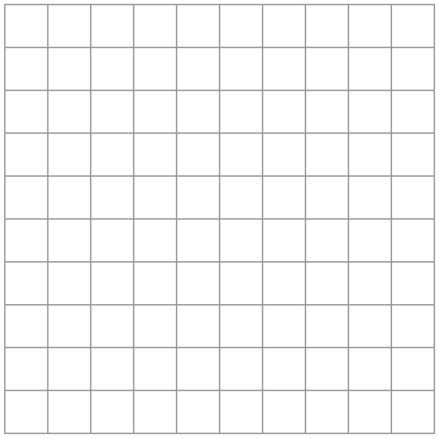
6. $4x + y > 6$



Problem 2 Systems of Linear Inequalities

The solution to a linear inequality in two variables is a half-plane, so the solution to a **system of two linear inequalities in two variables** is the portion of the coordinate plane where the half-planes overlap. For example:

1. Graph the solution to $y \geq 2x - 1$ in the grid.



2. On the same grid, graph the solution to $y \geq -3x + 2$.

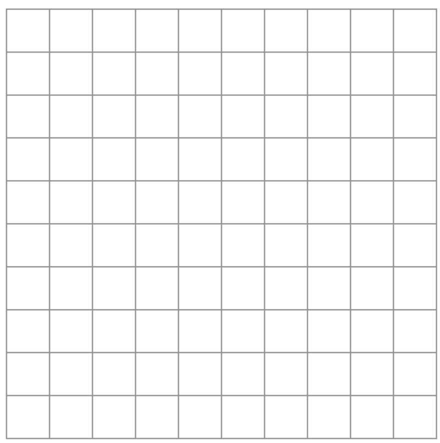
3. Do these solutions overlap? If so, shade this portion of the graph darker.

4. Choose a point in the region in which the solutions overlap. Substitute the ordered pair for this point into each of the inequalities. Does the ordered pair satisfy both inequalities?

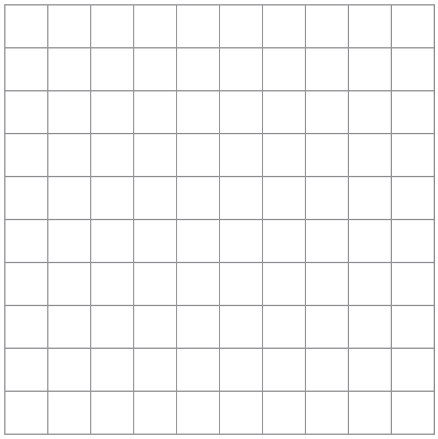
5. Choose a point from each of the other three regions created by the graphs of the inequalities. Substitute the ordered pair for each point into each of the inequalities. Do any of the ordered pairs satisfy both inequalities?

Solve each of the following systems of linear inequalities by graphing their solution set.

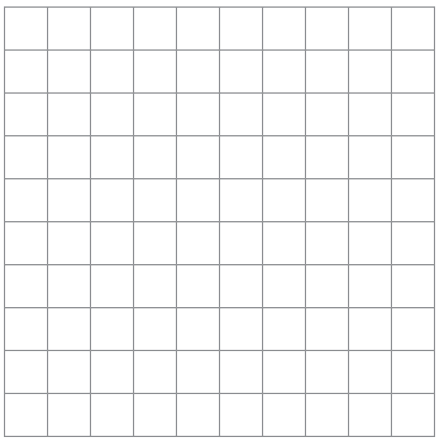
6. $\begin{cases} y \leq -2x \\ y > 3x + 5 \end{cases}$



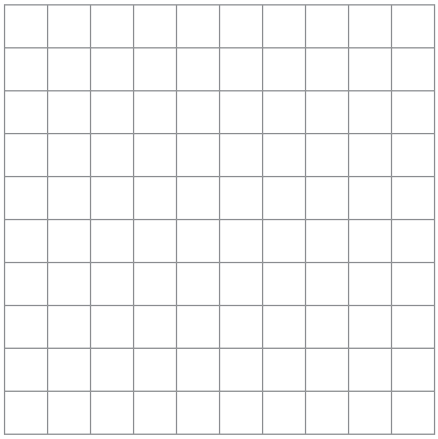
7. $\begin{cases} 2x + y > -2 \\ y > -x - 3 \end{cases}$



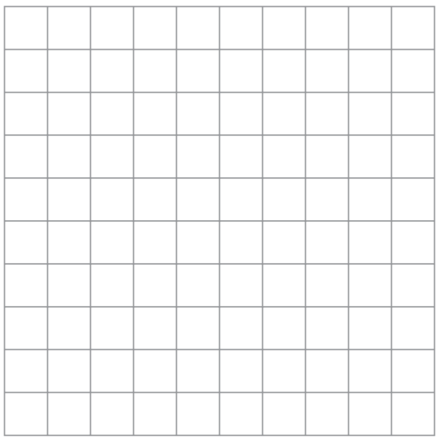
8. $\begin{cases} -2x \leq y \\ 2x + 5y \geq 10 \end{cases}$



9. $\begin{cases} y > 5 \\ 3x - 5y \geq 12 \end{cases}$



10. $\begin{cases} x + 7 < y \\ 4x - 5y \geq 40 \end{cases}$



Be prepared to share your work with another pair, group, or the entire class.

2.7 Three in Three or More

Solving Systems of Three or More Linear Equations in Three or More Variables

Objectives

In this lesson, you will

- Write a system of three linear equations in three variables.
- Solve a system of three linear equations in three variables using substitution.
- Solve a system of three linear equations in three variables using linear combinations/elimination.
- Solve a system of three linear equations in three variables using determinants.

Key Term

- 3×3 determinants

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Problem 1 More Ticket Problems

As a fund-raising event, your club sold tickets to a special viewing of one of the new X-Games films. The fund-raiser was successful: you sold out all 800 seats in the school's auditorium! You sold tickets at three different prices: $2.50 for children under 12 years old, $3.50 for youth 12 to 18 years old, and $5.00 for adults. The total amount of money taken in was $2937.50, and there were four times as many youth tickets as children's tickets sold. How many of each type of ticket was sold?

1. Define variables for each type of ticket.

a. Write an equation for the total number of tickets sold.

b. Write an equation for the total amount of money taken in.

c. Write a third equation relating the number of youth tickets sold to the number of children's tickets sold.

2. Write these three equations as a system.

This is a system of three linear equations in three variables. To solve a system like this, we need to reduce the system of three equations in three variables to a system of two equations in two variables, which we can then solve.

3. The equation $4c = y$ is already solved for a variable. So, substitute $4c$ for y in the other two equations to create a system of two equations in two variables.

4. Solve the resulting system by any of the three methods you already have learned.

5. How many of each type of ticket were sold?

Problem 2

Solve the following system using linear combinations. First, eliminate one variable from the first and second equations. Then, eliminate the same variable from the second and third equations. This will create a system of two equations in two variables.

1. $\begin{cases} 5x - 2y + z = 13 \\ 2x + y + z = 1 \\ -3x - 2y - z = 1 \end{cases}$

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Solve the following system using substitution. First, solve one equation for one variable. Then, substitute the resulting expression into each of the other two equations. This will create a system of two equations in two variables.

2. $\begin{cases} x + y + z = -2 \\ 2x - 3y + 2z = -14 \\ 4x + 3y - z = 5 \end{cases}$

3. To solve a system like this using Cramer's Rule, you need to have a method for evaluating a 3 × 3 determinant. Evaluate a **3 × 3 determinant** as follows:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix} = aei + bfg + cdh - ceg - afh - bdi$$

$\begin{cases} ax + by + cz = A \\ dx + ey + fz = B \\ gx + hy + iz = C \end{cases}$ where $a, b, c, d, e, f, g, h, i, A, B$, and C are constants, and x, y, and z are variables.

$$D = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \qquad D_x = \begin{vmatrix} A & b & c \\ B & e & f \\ C & h & i \end{vmatrix} \qquad D_y = \begin{vmatrix} a & A & c \\ d & B & f \\ g & C & i \end{vmatrix} \qquad D_z = \begin{vmatrix} a & b & A \\ d & e & B \\ g & h & C \end{vmatrix}$$

To determine the solution, calculate each of the following ratios:

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D} \qquad \text{where } D \neq 0$$

Notice that D is the determinant of the array coefficients. To calculate D_x, replace the coefficients of x with A, B, and C. To calculate D_y, replace the coefficients of y with A, B, and C. To calculate D_z, replace the coefficients of z with A, B, and C.

Solve the following system using Cramer's Rule:

$$\begin{cases} 2x + 3y + 2z = 10 \\ 3x - 4y + 5z = -25 \\ 4x + 5y + 3z = 19 \end{cases}$$

Problem 3

Solve each of the following systems using whichever method you prefer.

1. $\begin{cases} 1.2x + 0.5y + z = 4 \\ x - y + 5z = -19 \\ 3x - z = 19 \end{cases}$

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2. $$\begin{cases} \frac{2}{3}x + \frac{1}{6}y + \frac{1}{2}z = 2 \\ 4x + y + 2z = 8 \\ 3x + y - z = -1 \end{cases}$$

3. $$\begin{cases} 2x + y + z = -2 \\ 4x - 3y - 2z = -2 \\ -2x + 5y + 2z = 5 \end{cases}$$

Be prepared to share your work with another pair, group, or the entire class.